There are two aspects to this problem.

→ The CM of dumbell is uniformly accelerated.

→ WRT CM, the two particles perform SHM, starting from their respective "extreme" positions.

Now from question info, we can easily figure out two observations :→

(i) Since spring is compressed to max limit only once.

there is at most one complete cycle of oscillation.

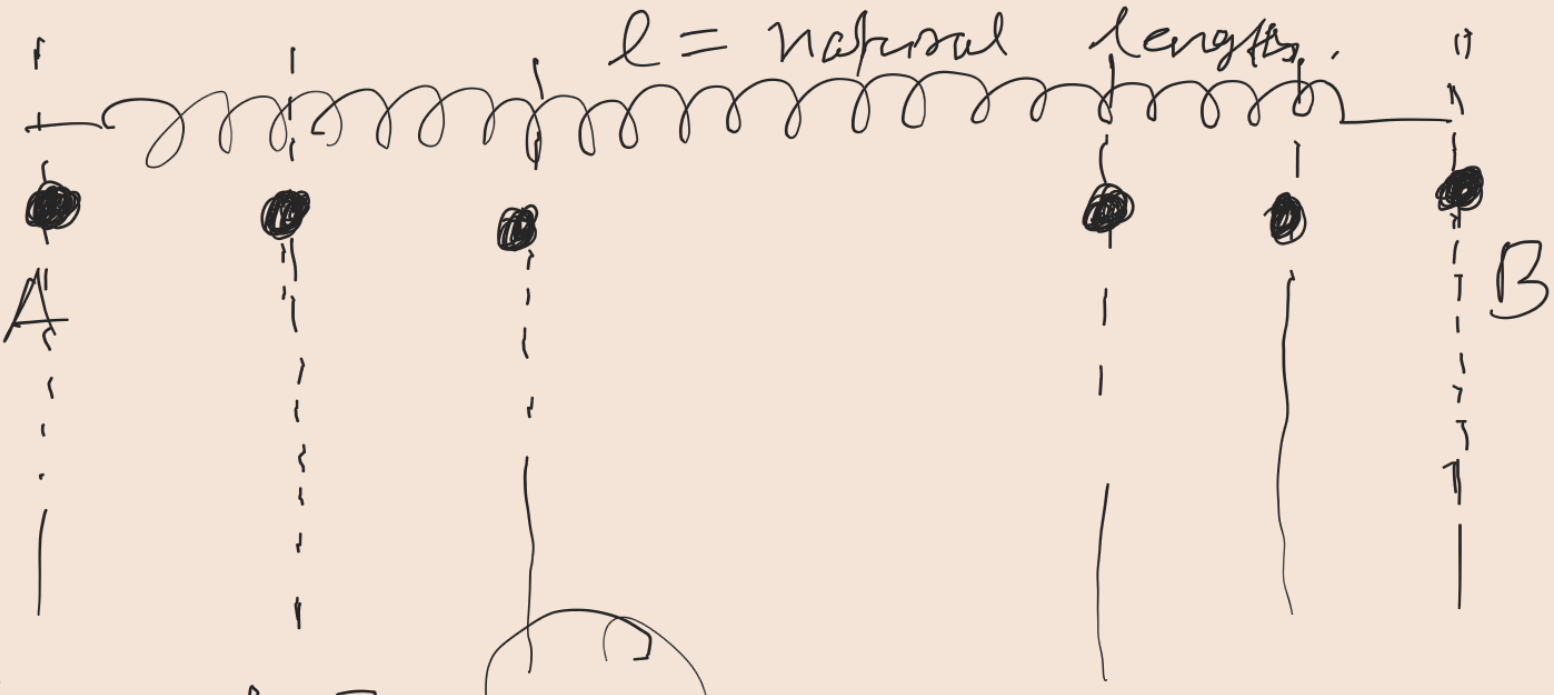
(ii) dumbell exit is with same velocity 'u'. This implies that exactly when 'B' exits the field, CM must have velocity 'u' in opposite direction to initial & vel. of both 'A' & 'B' wrt CM must be zero at that instant & spring must be in its natural length. That implies these

is exact one cycle of complete oscillation about CM. Now, time taken by 'B' in field is time taken by CM to reverse velocity vector. So,

$$v = -u + \left(\frac{qE}{2m}\right)t$$

$$t = \frac{4mu}{qE},$$ which is also the time period of one SHM cycle about CM.

Now, lets analyse the motion of 'A' & 'B' about CM.



$t = 0$ $t = \frac{\pi}{2\omega}$ $t = \frac{\pi}{\omega}$

$t = \frac{2\pi}{\omega}$ $t = \frac{3\pi}{2\omega}$ $V_{CM} = 0$.

$= \frac{4mu}{qE}$

We see that at $t = \frac{\pi}{\omega}$,

$V_{cm} = 0$ & $V_{A/cm} = 0$. Also,

beyond that V_{cm} & $V_{A/cm}$

both will be towards left.

Hence, from $0 \leq t \leq \frac{\pi}{\omega}$.

net displacement of 'A' is towards right side. Now $\omega = \sqrt{\frac{2k}{m}}$. At equilibrium, let compression of spring be 'x'. Then,

$$kx = \frac{qE}{2}. \quad \left[x = \frac{qE}{2k}\right].$$

Displacement of 'A' wrt CM

$$= \frac{x}{2} + \frac{x}{2} = x = \left(\frac{qE}{2k}\right).$$

Displacement of CM in this interval is $\frac{u^2}{2 \times \left(\frac{qE}{2m}\right)} = \left(\frac{mu^2}{qE}\right)$.

Net displacement of 'A'

$$= \frac{mu^2}{qE} + \frac{qE}{2k}.$$ Now

$$\frac{2}{\cancel{4}}\frac{mu}{qE} = \cancel{2}\pi\sqrt{\frac{m}{2k}}.$$ So,

$$\frac{(qE)^2}{2k} = \left(\frac{2u}{\pi}\right)^2 \times (m)$$ So,

net displacement of 'A' towards right is

$$\frac{mu^2}{qE}\left(1 + \frac{(qE)^2 \times 1}{2k \times mu^2}\right)$$

$$= \frac{mu^2}{qE}\left[1 + \frac{1}{2}\left(\frac{2u}{A}\right)^2 \times \frac{1}{\frac{1}{2}mu^2}\right]$$

$$= \frac{mu^2}{qE}\left[1 + \left(\frac{2}{A}\right)^2\right].$$

Since initial distance of 'A' from field boundary is 'l', which is spring natural length,

$$l \geq \frac{mu^2}{qE}\left\{1 + \left(\frac{2}{A}\right)^2\right\}.$$